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(Affiliated to CBSE up to +2 Level)

CLASS: X **SUB.: MATHS (NCERT BASED)** **DATE: 16-09-2020**

Ex 6.4

Q.1. Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Solution:

We have $\Delta ABC \sim \Delta DEF$

So, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

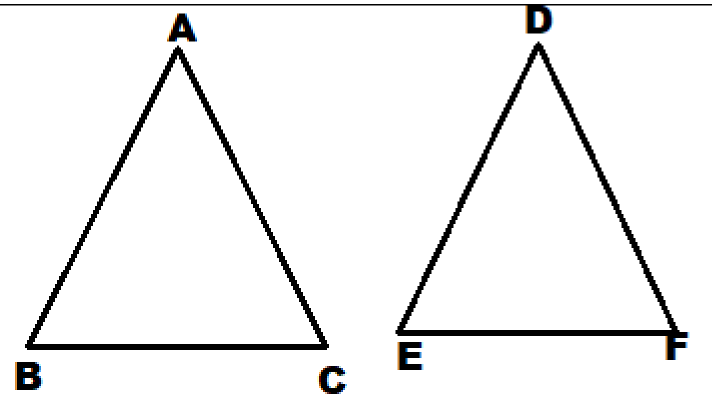
$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \frac{64}{121} = \frac{(BC)^2}{(15.4)^2} = \frac{BC^2}{237.16}$$

$$\Rightarrow 121BC^2 = 237.16 \times 64$$

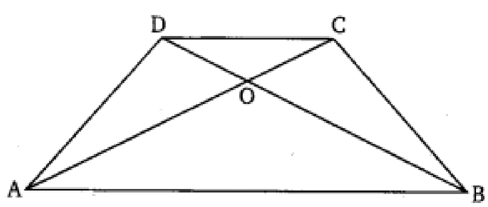
$$\Rightarrow BC^2 = \frac{15178.24}{121} = 125.44$$

$$\Rightarrow BC = \sqrt{125.44} = \mathbf{11.2\ cm.}$$



Q 2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Solution: In the figure below, a trapezium ABCD is shown, in which AB || DC and AB = 2DC. Its diagonals interest each other at the point O.



In ΔAOB and ΔCOD

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Corresponding angles]

$\therefore \Delta AOB \sim \Delta COD$ [By AA similarity]

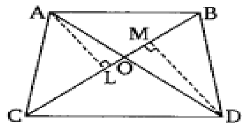
$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2} = \frac{(2 \times DC)^2}{DC^2}$$

$$= \frac{4 \times DC^2}{DC^2} = \frac{4}{1}$$

Hence, $ar(\Delta AOB) : ar(\Delta COD) = \mathbf{4 : 1}$

Q 3. **In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that:** $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$

Solution:



Draw $AL \perp BC$ and $DM \perp BC$

In ΔALO and ΔDMO, we have

$\angle ALO = \angle DMO$ [Each of 90°]

$\angle AOL = \angle DOM$ [Vertically opposite angles]

$\therefore \Delta ALO \sim \Delta DMO$ [By AA similarity]

$$\Rightarrow \frac{AL}{DM} = \frac{AO}{DO} \quad \ldots(i)$$

$$\text{Now} \quad \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM}$$

$$= \frac{AL}{DM} = \frac{AO}{DO} \quad \text{[From (i)]}$$

Hence, **proved**.

Q 4. **If the areas of two similar triangles are equal, prove that they are congruent.**

Solution:

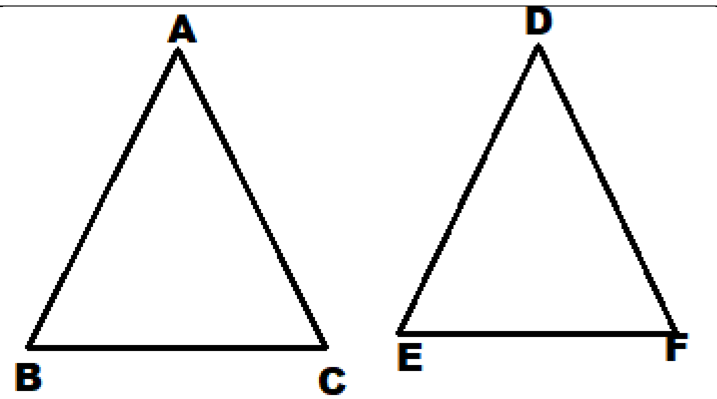
Let $\Delta ABC \sim \Delta DEF$

Then $ar(\Delta ABC) = ar(\Delta DEF)$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1 \Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{CA^2}{FD^2} = 1$$

$\therefore AB = DE, BC = EF, CA = FD$

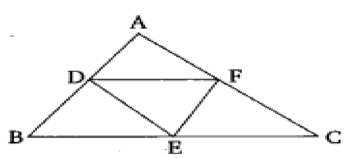
$\Rightarrow \Delta ABC \cong \Delta PQR$ Hence, **proved**.



5. **D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.**

Solution:

The given figure shows a ΔABC, in which D, E and F are the midpoints of sides AB, BC and CA respectively.



$\therefore DF = \frac{1}{2}BC$ [By Midpoint Theorem]

$DE = \frac{1}{2}CA$ [By Midpoint Theorem]

and $EF = \frac{1}{2}AB$ [By Midpoint Theorem]

$$\therefore \frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2}$$

$\Rightarrow \Delta DEF \sim \Delta ABC$

$$\text{Now,} \quad \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{DE^2}{AC^2}$$

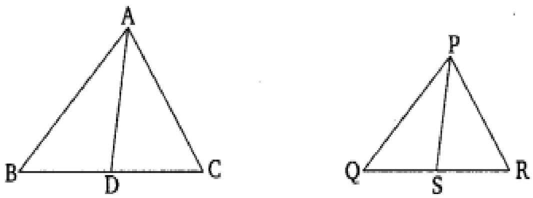
$$= \left(\frac{DE}{AC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Hence, the required ratio is **1 : 4**.

Q6.Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution:

Let AD and PS are the medians of ΔABC and ΔPQR respectively. [Corresponding angles] ...(i)



Then $\Delta ABC \sim \Delta PQR$ [Given]

$\Rightarrow \angle ABC = \angle PQR \quad \Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QS}$ [Median bisects the side]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QS}$

In ΔABD and ΔPQS,

$\angle ABD = \angle PQS$ [From (i)]

$\frac{AB}{PQ} = \frac{BD}{QS}$

$\Rightarrow \Delta ABD \sim \Delta PQS$ [By SAS similarity]

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PS}$ ...(ii)

Now $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ [By basic proportionality theorem]

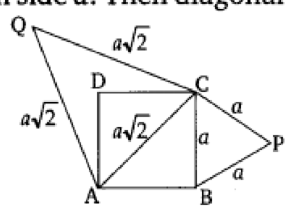
$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$ [From (ii)]

Hence, **proved**.

Q7.Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution:

Let ABCD be a square with side a. Then diagonal $AC = a\sqrt{2}$.



Since ΔBCP and ΔACQ are equilateral triangles, so they are similar.

$\therefore \frac{ar(\Delta BCP)}{ar(\Delta ACQ)} = \frac{BC^2}{AC^2} = \frac{a^2}{(a\sqrt{2})^2} = \frac{1}{2}$ ...(i)

$\Rightarrow ar(\Delta BCP) = \frac{1}{2} ar(\Delta ACQ)$ Hence, **proved**.

DO YOUR SELF

Q8.Tick the correct answer and justify

(i) ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(a) 2:1 **(b)** 1:2 **(c)** 4:1 **(d)** 1:4

(ii) Sides of two similar triangles are in the ratio 4:9. Areas of these triangles are in the ratio

(a) 2 :3 **(b)** 4:9 **(c)** 81:16 **(d)** 16:81